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Non-BPS D-branes and M-theory

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Abstract: A dual pair of supersymmetric string theories that involves an asymmetric orbifold and an orientifold of type II is considered. The D-branes of the orbifold theory (that were recently determined by Gutperle) are all non-BPS and do not carry any conserved gauge charges. It is shown that they carry non-trivial K-theory charges, and that they can be understood in terms of branes wrapping certain homology classes of the M-theory compactification. Using the adiabatic argument, dual partners of some of these non-BPS D-branes are proposed. The relations between these dual states are found to be in agreement with the M-theory description of the D-branes.

Keywords: M-Theory, D-branes, String Duality.

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Contents

1. Introduction

Recently much progress has been made in the study of non-BPS solitonic states in string theory [1]. A number of stable non-BPS D-branes have been constructed explicitly, most notably for certain orbifold theories [2]–[6], and type I (and type IA) [7]–[11]. All of these stable non-BPS D-branes can be elegantly characterized in terms of K-theory [12].

Non-BPS D-branes play a crucial role for our understanding of the strong-weak coupling dualities of certain supersymmetric string theories. While the dualities are most easily tested on the BPS spectrum of the theory, the duality map actually has to relate the whole spectrum of the two theories to one another. In particular, it is therefore interesting to understand what happens to some of the non-BPS states under this map. In some examples it was possible to identify the image of certain perturbative non-BPS states of one theory with non-BPS D-brane states of the dual theory [5, 7, 11]. In all of these cases, a crucial ingredient for the identification of these non-BPS states was the fact that they were the lightest states carrying a conserved gauge charge.

In this paper we consider a dual pair of supersymmetric string theories that relates an asymmetric orbifold to an orientifold of type II. The D-branes of the orbifold theory were recently determined by Gutperle [6]. All of these D-branes are in fact non-BPS and do not carry any conserved gauge charges. We confirm that these non-BPS D-branes are indeed stable by determining the corresponding K-theory groups which turn out to be $\mathbb{Z}_2$ in each case. (The situation is therefore similar to the case studied in [13], see also [14].) We then explain how these K-theory groups can be thought to arise from the cohomology of the corresponding M-theory compactification.[1] The relation between K-theory classes of R-R fields and cohomology in M-theory has recently received much attention [16, 17, 18]. In particular, it was shown in [16, 17] that the partition function for the R-R p-form fields of type IIA (that are classified by K-theory) agrees with the partition function for the p-form fields in M-theory (that are classified by a certain subset of cohomology). Some direct identifications between K-theory and cohomology classes were also found in [18].

The duality of the two string theories can be understood to originate from the duality of the corresponding theories before orbifolding or orientifolding, using the adiabatic argument of [19]. Some of the non-BPS D-branes of the orbifold theory have a direct interpretation in terms of brane anti-brane pairs in the original theory. We can therefore use the known duality relations for the theories before orbifolding and orientifolding to make a proposal for the duals of these non-BPS D-branes. These proposals can then be checked against the description of the non-BPS D-branes in terms of branes wrapping homology cycles of the M-theory compactification. We also make some speculations about the duals of some of the other non-BPS D-branes.

The paper is organised as follows. In section 2 we describe the different theories as well as their D-brane spectra and duality relations in detail. The latter are confirmed in section 3 by comparing the mass formulae for the BPS states in the different theories. In section 4 we explain how the D-branes can be understood in terms of M-branes wrapping homology cycles of the underlying M-theory compactification. In section 5 we make some proposals for the duals of certain non-BPS D-branes of the orbifold theory and perform various consistency checks. Section 6 contains some conclusions and open questions. We have included an appendix in which we give the details of the computation of the K-theory classes for the type II orbifold theories.

2. The duality relations and the D-brane spectrum

In this paper we want to consider the duality between the orbifold of type IIB

$$\text{IIB on} \qquad \frac{S^1}{(-1)^{F_L}\sigma_{1/2}}\,, \tag{2.1}$$

[1] Our argument is similar in spirit to the analysis of [15].

and the orientifold

$$\text{IIB on} \qquad \frac{S^1}{\Omega\sigma_{1/2}} \,. \tag{2.2}$$

Here $\sigma_{1/2}$ denotes the half-shift along an S^1 (which we shall take to lie in the x^9 direction). The orientifold theory is sometimes referred to as type $\tilde{\text{I}}$.[2] The duality between the orbifold and orientifold theory [20, 22] can be derived from the self-duality of type IIB using the adiabatic argument of [19].

We shall also consider the duality in eight dimensions obtained by compactifying these theories on an additional S^1 in the 8-direction. If we T-dualise both theories along x^8 we obtain IIA orbifold and orientifold theories, respectively,

$$\text{IIA on} \qquad \frac{S^1_8 \times S^1_9}{(-1)^{F_L}\sigma^9_{1/2}} \,, \qquad \text{and} \qquad \text{IIA on} \qquad \frac{S^1_8 \times S^1_9}{\Omega I_8 \sigma^9_{1/2}} \,, \tag{2.3}$$

where I_8 denotes the reflection in the x^8 direction. We shall sometimes refer to the orientifold theory as IA'. (This theory is not the same as what is usually called $\widetilde{\text{IA}}$, which is the T-dual of type $\tilde{\text{I}}$ along x^9.) The two IIA theories are S-dual as well. This can be seen from the point of view of M-theory [20] by compactifying M-theory on a Klein bottle times a circle, $(S^1_8 \times S^1_9)/\mathbb{Z}_2 \times S^1_{10}$, where $\mathbb{Z}_2$ acts as $I_8 \sigma^9_{1/2}$ together with a sign change in the three-form potential, S_{C^3}. From the point of view of the orientifold theory the coupling constant is proportional to $R_{10}^{3/2}$, while the coupling constant of the orbifold theory is proportional to $R_8^{3/2}$. The two theories are therefore related by an "8-10" flip. The relations between the various theories are sketched in figure 1.

More specifically, the relations between the moduli of the two IIB theories are given as

$$g_{IIB} = 4\, g_{\tilde{I}}^{-1} \,, \qquad R^i_{IIB} = \sqrt{2}\, g_{\tilde{I}}^{-1/2} R^i_{\tilde{I}} \,, \qquad G_{IIB} = 2\, g_{\tilde{I}}^{-1} G_{\tilde{I}} \,, \tag{2.4}$$

where R_i is any of the radii (including $i = 9$). Here and in the following we shall set $\alpha' = 1$. Together with the T-duality relations

$$\begin{aligned}
g_{IA'} &= \frac{g_{\tilde{I}}}{R^8_{\tilde{I}}} \,, & g_{IIA} &= \frac{g_{IIB}}{R^8_{IIB}} \,, \\
R^8_{IA'} &= \frac{1}{R^8_{\tilde{I}}} \,, & R^8_{IIA} &= \frac{1}{R^8_{IIB}} \,, \\
R^i_{IA'} &= R^i_{\tilde{I}} \,, \quad \text{for} \quad i \neq 8 \,, & R^i_{IIA} &= R^i_{IIB} \,, \quad \text{for} \quad i \neq 8 \,, \\
G_{IA'} &= G_{\tilde{I}} \,, & G_{IIA} &= G_{IIB} \,,
\end{aligned} \tag{2.5}$$

[2]In nine dimensions the moduli space of orientifold compactifications with sixteen supercharges has two components [18]: the usual type I vacuum for which both orientifold planes carry the same R-R charge, and the type $\tilde{\text{I}}$ theory for which the two R-R charges are opposite [20, 21]. In the latter case the overall R-R charge vanishes and there is no need to introduce D8-branes, thus leading to a trivial gauge group.

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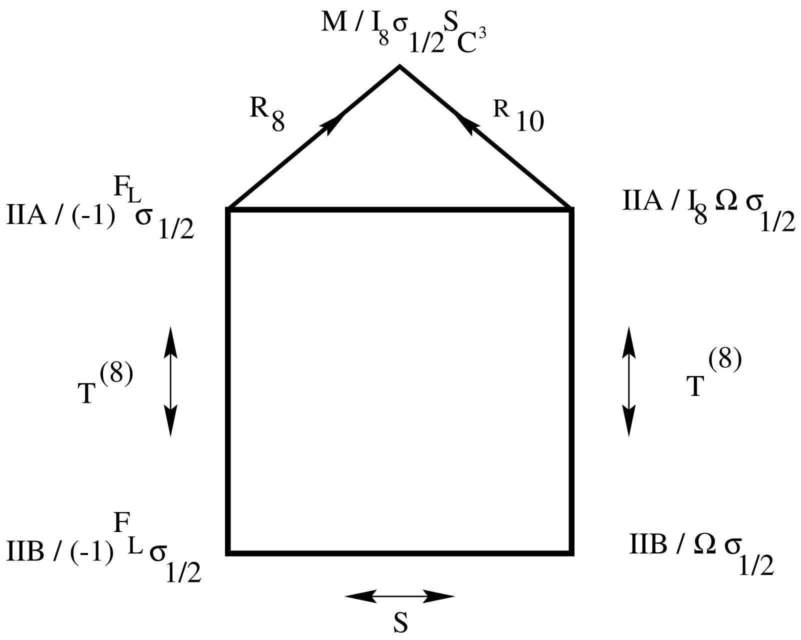

Figure 1: Duality relations in 8-dimensions.

this then implies the relations for the IIA moduli

$$
\begin{aligned}
g_{IIA} &= 2\sqrt{2}\,(g_{IA'})^{-1/2}(R^8_{IA'})^{3/2}\,, \\
R^8_{IIA} &= \frac{1}{\sqrt{2}}(g_{IA'})^{1/2}(R^8_{IA'})^{1/2}\,, \\
R^i_{IIA} &= \sqrt{2}(g_{IA'})^{-1/2}R^i_{IA'}(R^8_{IA'})^{1/2}\,, \qquad \text{for} \qquad i \neq 8\,, \\
G_{IIA} &= 2(g_{IA'})^{-1}R^8_{IA'}G_{IA'}\,.
\end{aligned}
\tag{2.6}
$$

These relations can be re-derived in terms of M-theory by setting

$$
R_8 = g_{IIA}^{2/3} = 2\frac{R^8_{IA'}}{g_{IA'}^{1/3}}\,, \qquad \text{and} \qquad R_{10} = \frac{1}{2}g_{IA'}^{2/3} = \frac{R^8_{IIA}}{g_{IIA}^{1/3}}\,, \tag{2.7}
$$

together with

$$
R_9 = \frac{R^9_{IIA}}{g_{IIA}^{1/3}} = \frac{R^9_{IA'}}{g_{IA'}^{1/3}}\,. \tag{2.8}
$$

Here, the radii without suffix are measured in M-theory. The formulae differ from those given by Hořava and Witten [23] by some factors of two, reflecting the fact that the moduli are effectively describing the double cover of the Klein bottle.

2.1 The D-brane spectrum

The orbifold theories have $(0,16)$ supersymmetry, while the supersymmetry of the orientifold theories is $(8,8)$. The latter theories therefore have BPS D-branes, while all the D-brane states of the orbifold theories are necessarily non-BPS. For the following it will be useful to summarise the D-brane spectra of the different theories.

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For the case of the IIB orbifold theory in nine dimensions the D-branes (and their boundary states) have been determined in [6]. There are two kinds of branes that are distinguished by the boundary condition in the compact 9-direction. Following the convention introduced in [24] we denote these D-branes as D(r,0) or D(r,1), where $r+1$ is the number of Neumann directions transverse to x^9, and the former branes are Dirichlet with respect to x^9, while the latter have a Neumann boundary condition. It follows from the analysis of [6] that both kinds of branes exist provided that r is even (odd) in IIA (IIB). Furthermore, for given such r, the two branes D(r,0) and D(r,1) can decay into one another depending on the size of the radius in the compact x^9 direction.

In the spirit of [12] the charges of these D-branes are classified by K-theory. Orbifold theories involving the action of $(-1)^{F_L}$ are described by the Hopkins groups $K_\pm$ [12], and the K-theory group that classifies D-brane charges in our context is therefore

$$K_\pm \left(S^{8-r} \times S^{2,0}, S^{2,0}\right) . \tag{2.9}$$

Here $r+1$ is the number of Neumann directions transverse to x^9. In writing (2.9) we have used the standard notation $\mathbb{R}^{p+q} \simeq \mathbb{R}^{p,q}$ to indicate that the (geometrical part of the) $\mathbb{Z}_2$-orbifold generator acts on the first p coordinates as -1, and we have denoted by $S^{p,q}$ the $p+q-1$ sphere in $\mathbb{R}^{p,q}$; we have also used the short hand notation $S^p = S^{0,p+1}$. On the circle $S^{2,0}$ the action of $\mathbb{Z}_2$ corresponds then precisely to the half-shift $\sigma_{1/2}$ (that accompanies $(-1)^{F_L}$ in the action of the orbifold generator). We have computed the groups (2.9) in appendix A. For the IIB orbifold we obtain

$$K_\pm \left(S^{8-r} \times S^{2,0}, S^{2,0}\right) = \begin{cases} \mathbb{Z}_2 \,, & \text{if } r \text{ is odd}\,, \\ 0\,, & \text{if } r \text{ is even}\,. \end{cases} \tag{2.10}$$

This is in agreement with the D-brane spectrum that was found by Gutperle in [6].[3] For the IIA orbifold we obtain similarly

$$K_\pm^{-1} \left(S^{8-r} \times S^{2,0}, S^{2,0}\right) = \begin{cases} \mathbb{Z}_2 \,, & \text{if } r \text{ is even}\,, \\ 0\,, & \text{if } r \text{ is odd}\,. \end{cases} \tag{2.11}$$

These K-theory calculations imply in particular that a single non-BPS D-brane is (topologically) stable, whereas an even number of such D-branes can decay into the vacuum. The $\mathbb{Z}_2$ nature of the charge can be understood from the fact that the D($r,0$) branes may be described in terms of brane anti-brane pairs in the theory before orbifolding [6].

The D-branes of the IIB orientifold theory in nine dimensions were determined in [10, 25] by computing the K-theory groups $\widetilde{KSC}(X)$; the results are summarised in table 1.

[3]Since the two branes D(r,0) and D(r,1) can decay into one another, they define the same K-theory class.

$D(r,s)$: $r =$	-1	0	1	2	3	4	5	6	7	8
$\widetilde{KSC}(S^{8-r})$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}$

Table 1: D-branes in the IIB orientifold in nine dimensions.

Upon compactifying on an additional circle and using T-duality (along x^8), the result for the IIA orientifold theory (in eight dimensions) can be derived from the above.

3. Comparison of BPS states

In order to check the duality relations (2.4) we shall next compare the mass formulae for the BPS states of the two IIB theories. We shall deal with the different classes of states in turn.

Massless states. In the NS-NS sector of the orbifold theory we have the massless bosonic states (with zero momentum) corresponding to the graviton, the $B_{\mu\nu}$ field and the dilaton; in the orientifold theory, the graviton and dilaton come from the NS-NS sector, while the $B_{\mu\nu}$ field arises in the R-R sector. Again, these are massless (and do not carry any momentum).

9-momentum states. In the untwisted NS-NS sector of the orbifold theory we have bosonic states that have even momentum,

$$(p_L, p_R) = \left(\frac{2m}{R^9_{IIB}}, \frac{2m}{R^9_{IIB}}\right). \tag{3.1}$$

Their mass in the orbifold theory is $M_{IIB} = 2|m|/R^9_{IIB}$, and in type $\tilde{\mathrm{I}}$ the mass is

$$M_{\tilde{I}} = \sqrt{2} g_{\tilde{I}}^{-1/2} \frac{2|m|}{\sqrt{2} g_{\tilde{I}}^{-1/2} R^9_{\tilde{I}}} = \frac{2|m|}{R^9_{\tilde{I}}}. \tag{3.2}$$

Thus these states are (closed string) momentum states in type $\tilde{\mathrm{I}}$ whose momentum is again even. In addition, there are bosonic states with odd momentum: in the orbifold theory these arise in the untwisted R-R sector (and therefore have $(-1)^{F_L}$ eigenvalue -1), and in the orientifold theory their eigenvalue under Ω is $\Omega = -1$.

9-winding states. The lightest bosonic state with 9-winding arises in the twisted NS-NS sector, and it is characterised by

$$(p_L, p_R) = \left(\frac{nR^9_{IIB}}{2}, -\frac{nR^9_{IIB}}{2}\right), \tag{3.3}$$

where $n \in \mathbb{Z}$. The IIB mass is $M_{IIB} = |n| R^9_{IIB}/2$, and in terms of type $\tilde{\mathrm{I}}$ this is

$$M_{\tilde{I}} = \sqrt{2} g_{\tilde{I}}^{-1/2} \frac{|n| \sqrt{2} g_{\tilde{I}}^{-1/2} R^9_{\tilde{I}}}{2} = g_{\tilde{I}}^{-1} |n| R^9_{\tilde{I}} . \tag{3.4}$$

This therefore represents a BPS D(0,1) brane in type $\tilde{\mathrm{I}}$, i.e. a D1-brane that wraps the S^1 in the 9-direction. This D-brane is invariant under the orientifold projection (as is also suggested by the K-theory calculation of table 1). Indeed, on the oscillator exponential of the boundary state, both Ω and $\sigma_{1/2}$ act trivially; for the case of a wrapped D1-brane, the ground state involves a sum over all 9-winding numbers, together with an integral over the transverse momenta. Ω maps $w_9 \mapsto -w_9$, and therefore leaves the sum invariant (provided the Wilson line is either zero or 1/2), and $\sigma_{1/2}$ acts trivially on winding states.[4]

There are 64 bosonic states of this type (together with 64 fermionic states that arise in the twisted NS-R sector) in the orbifold theory, and they form a short multiplet (of dimension 128) of the supersymmetry algebra. This agrees precisely with the degeneracy of the D1-brane BPS state.

8-momentum states. The analysis is fairly analogous to the case of 9-momentum: the states without momentum and winding in the 9-direction all come from the untwisted NS-NS sector, and they can have arbitrary integer 8-momentum. Using the same formula as before, these states correspond to (closed string) states in $\tilde{\mathrm{I}}$ that have arbitrary integer 8-momentum (and are invariant under Ω).

8-winding states. The case of winding in the 8-direction is more interesting. Again, the states without momentum and winding in the 9-direction all come from the untwisted NS-NS sector in the orbifold theory. Their momentum is now given by

$$(p_L, p_R) = \left(n R^8_{IIB}, -n R^8_{IIB} \right) , \tag{3.5}$$

and thus the mass is $M_{IIB} = |n| R^8_{IIB}$. In terms of type $\tilde{\mathrm{I}}$ this is

$$M_{\tilde{I}} = \sqrt{2} g_{\tilde{I}}^{-1/2} |n| \sqrt{2} g_{\tilde{I}}^{-1/2} R^8_{\tilde{I}} = 2 g_{\tilde{I}}^{-1} |n| R^8_{\tilde{I}} . \tag{3.6}$$

This corresponds to *two* D1-branes that wrap the 8-direction. This is necessary in order to have an orientifold invariant combination: the two D1-branes have to sit at the opposite points of the x^9-circle in order to be invariant under $\sigma_{1/2}$. In fact, this is one of the BPS D-branes that was discussed in [10].

There is one interesting lesson that can be drawn from this analysis that will prove useful later on. In the theory before orbifolding or orientifolding, S-duality exchanges the D1-brane with the fundamental string. One may have therefore thought

[4] A priori, there is an ambiguity in defining the $\mathbb{Z}_2$ shift action, that differs in the way the shift operator acts on winding states [19]. If the action of $\sigma_{1/2}$ in $\tilde{\mathrm{I}}$ was non trivial on winding states, the boundary state would not be invariant, and the BPS spectrum of the two theories would not agree.

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that the combination of two D1-branes in type $\tilde{\text{I}}$ theory should correspond, under S-duality, to the superposition

$$|w_8 = +1\,,\, x_9 = 0\rangle + |w_8 = +1\,,\, x_9 = \pi R^9_{IIB}\rangle \tag{3.7}$$

in the orbifold theory. (If we take these states to be the (lowest) GSO-invariant states in the NS-NS (NS-R) sector, these are indeed invariant under the orbifold projection.) In terms of a momentum-winding basis, this superposition would then be

$$\sum_{m\in\mathbb{Z}} \left| w_8 = +1\,,\, p_9 = \frac{2m}{R^9_{IIB}} \right\rangle . \tag{3.8}$$

However, this naive derivation does *not* agree with what we have found based on the analysis of the BPS masses: from that point of view, only the state with $p_9 = 0$ arises. We shall use the analogy with this situation to identify the dual of a non-BPS D-brane later on.

3.1 The relation to IIA and M-theory

Let us briefly comment on what these different states correspond to in the two IIA theories and in M-theory. A state with winding and momentum along x^8 and x^9 in the IIB orbifold theory has mass

$$M^2_{IIB} = \frac{n_9^2}{R_9^2} + m^2 R_9^2 + \frac{k^2}{R_8^2} + n_{10}^2 R_8^2 . \tag{3.9}$$

Under the duality map, this becomes

$$\begin{aligned} M^2_{\tilde{I}} &= \frac{n_9^2}{R_9^2} + 4\frac{m^2 R_9^2}{g^2} + \frac{k^2}{R_8^2} + 4\frac{n_{10}^2 R_8^2}{g^2} , \\ M^2_{IIA} &= \frac{n_9^2}{R_9^2} + m^2 R_9^2 + k^2 R_8^2 + \frac{n_{10}^2}{R_8^2} , \\ M^2_{IA'} &= \frac{n_9^2}{R_9^2} + 4\frac{m^2 R_8^2 R_9^2}{g^2} + k^2 R_8^2 + 4\frac{n_{10}^2}{g^2} , \end{aligned} \tag{3.10}$$

where we have written all masses in terms of the moduli of the respective theories. The BPS states are then identified as follows: the IIB 9-momentum states (with even momentum) are mapped to IIA 9-momentum states (with even momentum), which are 9-momentum states (with even momentum) in IA'. A 9-winding state with half-integer winding (coming from the twisted sector) is mapped to a single IA' D2-brane wrapping the 8- and 9-directions. 8-momentum states in IIB map to 8-winding states in IIA and 8-winding states in IA'. On the other hand, 8-winding states in IIB have IA'-mass

$$M_{IA'} = \frac{2n}{g} , \tag{3.11}$$

and therefore correspond to *two* D0-branes; this is again required in order to obtain an orientifold invariant configuration.

All of these states can be thought of as arising from M-theory where they correspond to states with mass

$$M_M^2 = \frac{n_9^2}{R_9^2} + m^2 R_8^2 R_9^2 + k^2 R_8^2 R_{10}^2 + \frac{n_{10}^2}{R_{10}^2} \,. \tag{3.12}$$

The relevant states are KK-momentum states with momentum along 9 and 10, and membrane winding states, respectively. In the untwisted sector of the orbifold theory, all quantum numbers are integers, but in the twisted sector we also have states for which the 9-winding number m is half-integer. From the point of view of M-theory, these states will therefore define new kinds of *twisted membrane* states. Given our limited understanding of M-theory orbifolds, we do not know how to derive the existence of these states directly in terms of M-theory. (In particular, they do not seem to be necessary for the cancellation of gravitational anomalies as in [23]; the situation is therefore similar to what was found in [26].) However, given that the relevant states in the orbifold theory are BPS, these states must be present in the M-theory spectrum.

4. K-theory charges from M-theory cohomology

Recently it has been proposed that K-theory fluxes in type IIA string theory may be related to certain M-theory cohomology classes [16, 17, 18]. Before proceeding, we want to explore a related question for the case at hand, namely whether the K-theoretic D-brane charges of the IIA orbifold theory can be understood in terms of M-theory cohomology. This is of particular interest in our case since the D-brane charges are all pure torsion.

Because of Poincaré duality (see (4.3) below for the relation in our case) the corresponding D-branes should be described in terms of M-branes wrapping suitable homology cycles of the M-theory compactification. Since all the D-branes of the IIA orbifold are non-BPS this amounts to identifying the M-theory lift of these non-BPS D-brane states. Non-BPS states in M-theory have been discussed before in [27, 28], where they were constructed out of M-brane anti-M-brane pairs by tachyon condensation, and in [15] where they were obtained by lifting non-BPS configurations in a IIA orientifold theory to 11 dimensions; our analysis here will be similar in spirit to the latter approach.

For the theory in question M-theory is compactified on a Klein bottle (times the 10-circle that does not play a role in the following since it is common to both M-theory and the IIA orbifold). Since the Klein bottle $\mathbf{K}$ is not orientable there are two kinds of (co)homologies: normal integral (co)homology and "twisted" (co)homology which takes coefficients in $\widehat{\mathbb{Z}}$. Here $\widehat{\mathbb{Z}}$ is the $\mathbb{Z}_2$-module where $\mathbb{Z}_2$ acts by the non-trivial representation (for details on twisted (co)homologies see for example [29]). The normal integral homology of $\mathbf{K}$ is given by

$$H_n(\mathbf{K},\mathbb{Z}) = \begin{cases} \mathbb{Z}\,, & n=0\,,\\ \mathbb{Z}\oplus\mathbb{Z}_2\,, & n=1\,,\\ 0\,, & n\geq 2\,, \end{cases} \tag{4.1}$$

and the corresponding integral cohomology is

$$H^n(\mathbf{K},\mathbb{Z}) = \begin{cases} \mathbb{Z}\,, & n=0\,,\\ \mathbb{Z}\,, & n=1\,,\\ \mathbb{Z}_2\,, & n=2\,,\\ 0\,, & n\geq 3\,. \end{cases} \tag{4.2}$$

The "twisted" (co)homology $\widehat{H}^{(*)}_{*}(\mathbf{K},\widehat{\mathbb{Z}})$ can be obtained from a twisted version of Poincaré duality[5]

$$\widehat{H}_n(\mathbf{K},\widehat{\mathbb{Z}}) = H^{d-n}(\mathbf{K},\mathbb{Z})\,, \qquad \text{and} \qquad \widehat{H}^n(\mathbf{K},\widehat{\mathbb{Z}}) = H_{d-n}(\mathbf{K},\mathbb{Z})\,, \tag{4.3}$$

where, in our case, $d=2$.

Under the action of the M-theory orbifold, the 3-form changes sign, and therefore the membrane reverses its orientation. This implies that the membrane can only wrap on "twisted" homology cycles $\widehat{H}_n(\mathbf{K},\widehat{\mathbb{Z}})$. It follows from (4.3) and (4.2) that there are two $\mathbb{Z}$-cycles for $n=1$ and $n=2$, and a $\mathbb{Z}_2$ cycle for $n=0$. Wrapping the membrane around the 2-cycle gives rise to a fundamental string with 9-winding in the orbifold theory, whereas the membrane that is wrapped around the 1-cycle produces a fundamental string state without 9-winding. Both of these states are indeed BPS (and carry $\mathbb{Z}$ charge). On the other hand, the $\mathbb{Z}_2$ 0-cycle corresponds to the non-BPS D(2,0) brane of the IIA orbifold theory. This brane can be thought of as coming from a D2-brane anti-brane pair in the theory before orbifolding (where the two branes sit at opposite points of the 9-circle). Naively this would seem to lift to a M2-brane anti-brane pair; however, the above description in terms of a twisted homology class suggests that the configuration actually comes from a single M2-brane that "wraps" this twisted cycle.

The other extended object in M-theory is the M5-brane that is unaffected by the orbifold action, and that should therefore only wrap around untwisted cycles $H_n(\mathbf{K},\mathbb{Z})$. It follows from (4.1) that there are two $\mathbb{Z}$-cycles for $n=0$ and $n=1$, as well as a $\mathbb{Z}_2$ cycle for $n=1$. If we "wrap" the M5-brane around the 0-cycle, we obtain the NS 5-brane in the orbifold theory, whilst wrapping the M5-brane around the $\mathbb{Z}$ 1-cycle gives rise to a NS 5-brane that wraps the 9-direction. (This is an allowed configuration since the NS 5-brane is invariant under $(-1)^{F_L}$.) On the other

[5] This notion of "twisting" has nothing to do with the "twisted" sectors of the orbifold.

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hand, the $\mathbb{Z}_2$ 1-cycle corresponds to the non-BPS D(4,0) brane of the orbifold theory. Again this brane can be thought of as coming from a D4 anti-D4-brane pair in the theory before orbifolding which naively lifts to M5 anti-M5-brane pair in M-theory. However, the above homology analysis suggests again that one can think of this as a single M5-brane wrapping the $\mathbb{Z}_2$ homology 1-cycle.

5. Identifying non-BPS states

In this section we shall attempt to identify the non-BPS D-brane states of the IIB orbifold theory with non-BPS states of the dual type $\tilde{\mathrm{I}}$ theory. We shall also explain how our proposals tie in with the description of the T-dual non-BPS states of the IIA orbifold in terms of M-theory.

As we have reviewed in section 2, there are two types of D-branes states in the IIB orbifold theory that can decay into one another depending on the radius of the circle in the 9-direction [6]

$$(2r-1,0) \longleftrightarrow (2r-1,1)\,. \tag{5.1}$$

The branes of the form $\mathrm{D}(2r-1,0)$ can be thought to consist of a $\mathrm{D}(2r-1,0)$ brane of type IIB at $x^9 = a$ together with a $\mathrm{D}(2r-1,0)$ anti-brane at $x^9 = a + \pi R^9$. This is an orbifold invariant configuration since $(-1)^{F_L}$ maps a brane to an anti-brane. The boundary state of this non-BPS D-brane only involves the untwisted sector as should be expected from this interpretation. On the other hand, the boundary state of the $\mathrm{D}(2r-1,1)$ brane has non-trivial components in the untwisted NS-NS as well as the twisted R-R sector; its interpretation in terms of type IIB branes is therefore not clear.

5.1 The D(3,0)-brane

There is one non-BPS D-brane of the orbifold theory that can be quite easily identified with a non-BPS state of type $\tilde{\mathrm{I}}$: this is the D(3,0) brane that has an interpretation in terms of a brane anti-brane pair of the type IIB theory. We know that in the theory before orbifolding and orientifolding, the D3-brane of type IIB is self-dual. Thus we should expect that the D(3,0) brane of the orbifold theory corresponds to a D-brane in type $\tilde{\mathrm{I}}$ that can be made out of a D3-brane at $x^9 = a$ together with an anti-D3-brane at $x^9 = a + \pi R^9$. The corresponding boundary state describes in fact precisely the D(3,0) brane of type $\tilde{\mathrm{I}}$ that was constructed in [10]. Thus we have identified

$$\begin{array}{ccc} \text{IIB-orbifold} & & \text{Type } \tilde{\mathrm{I}}\,, \\ \mathrm{D}(3,0) & \longleftrightarrow & \mathrm{D}(3,0)\,. \end{array} \tag{5.2}$$

Both of these D-branes are $\mathbb{Z}_2$ charged, and the reason is the same in both cases: if we have two such branes, we can move the D3-brane of one combination to come close to the anti-D3-brane of the other, and they annihilate.

Suppose the D(3,0) does not wrap the 8-direction. Under T-duality (along x^8), the D(3,0) brane then becomes a D(4,0) brane in the IIA orbifold theory (that wraps the 8-direction of the orbifold theory, i.e. the 10-direction of M-theory). As we have explained before, this brane lifts in M-theory to the M5-brane wrapping the $\mathbb{Z}_2$ 1-cycle of the Klein bottle (as well as the 10-direction). Under the "8-10 flip" this then becomes a D4-brane in the IA' theory (since the M5-brane wraps along the 10-direction), and therefore, under T-duality, a D3-brane in $\tilde{\mathrm{I}}$. This agrees with (5.2).

On the other hand, if the D(3,0) does wrap the 8-direction, then under T-duality (along x^8) the D(3,0) brane becomes a D(2,0) brane in the IIA orbifold (that does not wrap the 10-direction). This brane lifts to the M2-brane in M-theory that corresponds to the twisted 0-cycle of the Klein bottle. Under the "8-10 flip" this then becomes again a D2-brane in IA', and thus a D3-brane in $\tilde{\mathrm{I}}$. This also agrees with (5.2).

5.2 The D(1,0)-brane

Next let us consider the D(1,0) brane of the IIB orbifold theory. As we have explained before, this brane can be thought to consist of a D1-brane at $x^9 = 0$, together with an anti-D1-brane at $x^9 = \pi R^9$. Under S-duality, one should therefore expect that this state corresponds to something like

$$|w_8 = +1\,,\, x_9 = 0\rangle + |w_8 = -1\,,\, x_9 = \pi R^9\rangle \tag{5.3}$$

in type $\tilde{\mathrm{I}}$. Here we have assumed that the D(1,0) brane wraps along the x^8 direction.

The above argument is fairly analogous to the one we put forward for the BPS 8-winding states. In that case we saw that the actual dual state was not (5.3), but rather the lowest momentum component; so we propose now that the dual of the D(1,0) brane of the orbifold theory is the superposition

$$|w_8 = +1\,,\, p_9 = 0\rangle + \epsilon |w_8 = -1\,,\, p_9 = 0\rangle\,. \tag{5.4}$$

Here $|w_8, p_9\rangle$ describes the momentum and winding of the 128 bosonic states that are the lowest GSO-invariant states in the NS-NS and R-R sector, and similarly for the fermions. In order for the above superposition to be invariant under the orientifold projection, we have to choose ϵ to be the eigenvalue of the corresponding state under the action of Ω, i.e. $\epsilon = +1$ if the ground state describes the graviton and dilaton in the NS-NS sector or the $B_{\mu\nu}$ state in the R-R sector, and $\epsilon = -1$ otherwise. There are 64 states for which $\Omega = \epsilon = +1$, and 64 states for which $\Omega = \epsilon = -1$. Together with the 128 fermions these states form a long (non-BPS) multiplet of the supersymmetry algebra. (It is clear that they form one long multiplet rather than two short multiplets since they are not BPS; indeed, under the duality map their mass in the orbifold theory is that of a D-brane, and we know that none of the D-branes of the orbifold theory are BPS.)

One might wonder whether the states in (5.4) are actually stable, or whether they can decay to states with $w_8 = p_9 = 0$. At $w_8 = p_9 = 0$ there are only 64 physical boson states since only the states with $\Omega = +1$ survive the orbifold projection. These 64 states (together with their fermions) form a short multiplet of the supersymmetry algebra and these states are indeed BPS. Thus it would seem that the states with $\epsilon = +1$ can decay to $w_8 = p_9 = 0$, but that this is not possible for the states with $\epsilon = -1$. However, since multiplets must decay as whole multiplets this means that the whole long multiplet cannot decay in this way.

We thus propose that the dual of the non-BPS D(1,0) brane of the IIB orbifold theory is the state in (5.4),

$$\begin{array}{ccc} \text{IIB-orbifold} & & \text{Type } \tilde{\mathrm{I}}\,, \\ \mathrm{D}(1,0) & \longleftrightarrow & w^i = \pm 1\,, \end{array} \tag{5.5}$$

where w^i is the winding number in the (compact) direction along which the D(1,0) brane wraps.

As before, we can check whether this proposal makes sense in the T-dual IIA picture. Under T-duality, the non-BPS D(1,0) brane (that wraps x^8) maps to the non-BPS D(0,0) brane of the IIA orbifold. The D(0,0) brane can be thought of as a D0-brane anti-brane pair of the theory before orbifolding (where the two branes are at opposite points of the 9-circle). In M-theory these correspond to a combination of states that have (positive and negative) 10-momentum. Under the "8-10" flip we therefore obtain the state in IA'

$$|p_8 = 1\,,\, p_9 = 0\rangle + \epsilon |p_8 = -1\,,\, p_9 = 0\rangle\,. \tag{5.6}$$

This is precisely the T-dual (along x^8) of (5.4).

Similarly, the T-dual of the D(1,0) brane (that wraps x^7, say, but not x^8) is the non-BPS D(2,0) brane of the IIA orbifold (that wraps x^7 and x^8, i.e. the 10-direction of M-theory). In M-theory this lifts to the M2-brane wrapping the 7- and 10-direction (as well as the twisted 0-cycle of the Klein bottle). Under the "8-10 flip" this then becomes a fundamental string state in IA' that has winding along the 7-direction. This is again in agreement with (5.5).

5.3 A speculation

As is explained in [6] the D(3,0) brane of the orbifold theory can decay, for sufficiently small R^9_{IIB}, to a D(3,1) non-BPS D-brane. On the other hand, the dual D(3,0) brane of type $\tilde{\mathrm{I}}$ can decay, for sufficiently small $R^9_{\tilde{I}}$, into the non-BPS $\mathrm{D}(3,1)$ brane of type $\tilde{\mathrm{I}}$. It follows from (2.4) that $R^9_{\tilde{I}}$ is proportional to R^9_{IIB}, and therefore that the regimes of stability are at least qualitatively related to one another. This then suggests that we can also identify

$$\begin{array}{ccc} \text{IIB-orbifold} & & \text{Type } \tilde{\mathrm{I}}\,, \\ \mathrm{D}(3,1) & \longleftrightarrow & \mathrm{D}(3,1)\,. \end{array} \tag{5.7}$$

However, it should be clear that this argument is somewhat unreliable since the actual relation between the moduli is $R^9_{IIB} = \sqrt{2} g_{\tilde{I}}^{-1/2} R^9_{\tilde{I}}$, and thus R^9_{IIB} can also become large if $g_{\tilde{I}}$ becomes small (for arbitrary $R^9_{\tilde{I}}$). Furthermore, in the regime in which the M-theory description (that interpolates between the two dual string theories) is valid we have $R_8, R_9, R_{10} \gg 1$, and therefore both R^9_{IIB} and $R^9_{\tilde{I}}$ are large; in particular, this suggests that in the M-theory regime, the $\mathrm{D}(r,1)$ brane may always be unstable to decay into the $\mathrm{D}(r,0)$ brane. If this is the case we cannot simply identify the dual of the $\mathrm{D}(r,1)$ brane by keeping track (as we vary the moduli to go from weak to strong coupling) of the lightest state with certain properties.

6. Conclusions and outlook

In this paper we have analysed the duality between the asymmetric orbifold $\mathrm{IIB}/(-1)^{F_L}\sigma_{1/2}$ and the orientifold $\mathrm{IIB}/\Omega\sigma_{1/2}$, as well as their T-dual IIA theories. We have shown that the non-BPS D-branes of the orbifold theory that had been constructed by Gutperle [6] carry $\mathbb{Z}_2$ K-theory charge (but do not carry any conserved gauge charge). We have also explained how these D-branes can be understood in terms of homology cycles of M-theory. Among other things, this sheds some light on non-BPS states in M-theory; this is of interest since the tachyon condensation arguments that underlie for example [27] are at present not very well understood.

Using the fact that the duality can be understood to originate from a duality of the theories before orbifolding and orientifolding, we have made a proposal for the duals of the non-BPS D-branes of the orbifold theory. We have also shown that these proposals are in agreement with their M-theory interpretation. Since the non-BPS states do not carry any conserved gauge charges, this identification of non-BPS states goes beyond what had been achieved before.

It would be interesting to perform a similar analysis for the case of the CHL string [30] which corresponds to M-theory on a Möbius strip. It would also be illuminating to understand how the (non-BPS) D-branes of the type IA theory can be understood in terms of M-theory homology cycles, and in particular, how the gauge charges of the D-string [11] arise from this point of view. Finally, it may be interesting to study fluxes in these models along the lines of [18].

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A. K-theory analysis for the orbifold theory

In this appendix we shall compute the groups (2.9); related calculations have been done before in [24, 31]. By a result of Hopkins (not published, but see [12]), $K_\pm$ can be related to the equivariant K-theory groups as

$$K_{\pm,\mathrm{cpt}}(X) = K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(X \times \mathbb{R}^{1,0}\right), \tag{A.1}$$

where the suffix "cpt" denotes K-theory groups with compact support. Since $S^{2,0}$ is a retract of $S^{8-r} \times S^{2,0}$ we can write

$$K_\pm\left(S^{8-r} \times S^{2,0}, S^{2,0}\right) \oplus K_{\pm,\mathrm{cpt}}\left(S^{2,0}\right) = K_{\pm,\mathrm{cpt}}\left(S^{8-r} \times S^{2,0}\right). \tag{A.2}$$

Together with the relation (A.1) this implies

$$K_\pm\left(S^{8-r} \times S^{2,0}, S^{2,0}\right) \oplus K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0} \times \mathbb{R}^{1,0}\right) = K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{8-r} \times S^{2,0} \times \mathbb{R}^{1,0}\right). \tag{A.3}$$

Further it follows from the Künneth formula [32] that

$$K^m_{\mathbb{Z}_2,\mathrm{cpt}}\left(X \times S^{2n}\right) = K^m_{\mathbb{Z}_2,\mathrm{cpt}}(X)^{\oplus 2} \tag{A.4}$$

and

$$K^m_{\mathbb{Z}_2,\mathrm{cpt}}\left(X \times S^{2n+1}\right) = K^m_{\mathbb{Z}_2,\mathrm{cpt}}(X) \oplus K^{m-1}_{\mathbb{Z}_2,\mathrm{cpt}}(X). \tag{A.5}$$

Putting all of this together we arrive at

$$K_\pm\left(S^{8-r} \times S^{2,0}, S^{2,0}\right) = \begin{cases} K_{\mathbb{Z}_2,\mathrm{cpt}}(S^{2,0} \times \mathbb{R}^{1,0}), & \text{if } r \text{ is odd}, \\ K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}(S^{2,0} \times \mathbb{R}^{1,0}), & \text{if } r \text{ is even}. \end{cases} \tag{A.6}$$

It therefore remains to compute the K-theory groups $K^*_{\mathbb{Z}_2,\mathrm{cpt}}(S^{2,0} \times \mathbb{R}^{1,0})$.[6] These can be computed by using the long exact sequence

$$\begin{aligned} \cdots \widetilde{K}^{-2}\left(S^{2,0}\right) &\longrightarrow K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0} \times \mathbb{R}^{1,0}\right) \longrightarrow K^{-1}_{\mathbb{Z}_2}\left(S^{2,0}\right) \longrightarrow K^{-1}\left(S^{2,0}\right) \\ &\longrightarrow K_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0} \times \mathbb{R}^{1,0}\right) \longrightarrow \widetilde{K}_{\mathbb{Z}_2}\left(S^{2,0}\right) \longrightarrow \widetilde{K}\left(S^{2,0}\right) \longrightarrow \cdots, \end{aligned} \tag{A.7}$$

where $\widetilde{K}(X)$ denotes the reduced K-theory group, $K(X) = \widetilde{K}(X) \oplus \mathbb{Z}$. Next we observe that $S^{2,0}/\mathbb{Z}_2 = \mathbb{R}\mathbf{P}^1$, and therefore

$$K^n_{\mathbb{Z}_2}\left(S^{2,0}\right) = K^n\left(\mathbb{R}\mathbf{P}^1\right). \tag{A.8}$$

Given the results of [33, 34] it then follows that

$$\widetilde{K}^{2n}_{\mathbb{Z}_2}\left(S^{2,0}\right) = 0, \qquad \text{and} \qquad K^{2n+1}_{\mathbb{Z}_2}\left(S^{2,0}\right) = \mathbb{Z}. \tag{A.9}$$

[6] We can also compute these by noting that the compactification of the canonical line-bundle on $\mathbb{R}\mathbf{P}^1$, $S^{2,0} \times \mathbb{R}^{1,0}/\mathbb{Z}_2$, is $\mathbb{R}\mathbf{P}^2$. The (complex) K-theory groups for $\mathbb{R}\mathbf{P}^n$ have been computed in [33, 34] and are given by $\widetilde{K}^m(\mathbb{R}\mathbf{P}^n) = \delta_{m,2l}\mathbb{Z}_{2^{[n/2]}}$ where $l \in \mathbb{Z}$. It therefore follows that $K_{\mathbb{Z}_2,\mathrm{cpt}}(S^{2,0} \times \mathbb{R}^{1,0}) = \widetilde{K}(\mathbb{R}\mathbf{P}^2) = \mathbb{Z}_2$. Similarly, we obtain $K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}(S^{2,0} \times \mathbb{R}^{1,0}) = 0$.

This implies that $K^n(S^{2,0}) = \mathbb{Z}$ for all n (since the reduced and unreduced K-theory groups differ by $\mathbb{Z}$ for n even only). Putting these results in the long exact sequence (A.7) we then find

$$0 \longrightarrow K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0}\times\mathbb{R}^{1,0}\right) \longrightarrow \mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z} \longrightarrow K_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0}\times\mathbb{R}^{1,0}\right) \longrightarrow 0\,. \quad (A.10)$$

The map between $K^{-1}_{\mathbb{Z}_2}(S^{2,0}) = K^{-1}(\mathbb{R}\mathbf{P}^1)$ and $K^{-1}(S^{2,0})$ is multiplication by 2 because the map between S^1 and $\mathbb{R}\mathbf{P}^1$ has degree 2. It now follows from (A.10) that

$$K_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0}\times\mathbb{R}^{1,0}\right) \cong \mathbb{Z}_2 \quad (A.11)$$

and

$$K^{1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{2,0}\times\mathbb{R}^{1,0}\right) \cong 0\,. \quad (A.12)$$

Together with (A.6) we therefore arrive at the result

$$K_{\pm}\left(S^{8-r}\times S^{2,0}, S^{2,0}\right) = \begin{cases} \mathbb{Z}_2\,, & \text{if } r \text{ is odd}\,,\\ 0\,, & \text{if } r \text{is even}\,. \end{cases} \quad (A.13)$$

This reproduces (2.10) and is in agreement with the D-brane spectrum that was found by Gutperle in [6].

The K-theory groups characterizing the D(r,s) branes for the orbifold of the IIA theory by $(-1)^{F_L}\sigma_{1/2}$ are

$$K^{-1}_{\pm}\left(S^{8-r}\times S^{2,0}, S^{2,0}\right). \quad (A.14)$$

Using the identity that follows from (A.1)

$$K^{-1}_{\pm,\mathrm{cpt}}(X) = K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(X\times\mathbb{R}^{1,1}\right), \quad (A.15)$$

and similar arguments as above, we obtain an analogue of (A.6) where now $\mathbb{R}^{1,0}$ is replaced by $\mathbb{R}^{1,1}$. We can then use the suspension isomorphism

$$K^{-1}_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{n,0}\times\mathbb{R}^{1,1}\right) = K_{\mathbb{Z}_2,\mathrm{cpt}}\left(S^{n,0}\times\mathbb{R}^{1,0}\right) \quad (A.16)$$

and conclude that

$$K^{-1}_{\pm}\left(S^{8-r}\times S^{2,0}, S^{2,0}\right) = \begin{cases} \mathbb{Z}_2\,, & \text{if } r \text{ is even}\,,\\ 0\,, & \text{if } r \text{ is odd}\,. \end{cases} \quad (A.17)$$

This reproduces (2.11).

References

[1] A. Sen, *Non-BPS states and branes in string theory*, `hep-th/9904207`.

[2] A. Sen, *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [`hep-th/9805019`].

[3] O. Bergman and M.R. Gaberdiel, *Stable non-BPS D-particles*, *Phys. Lett.* **B 441** (1998) 133 [hep-th/9806155].

[4] A. Sen, *BPS D-branes on non-supersymmetric cycles*, *J. High Energy Phys.* **12** (1998) 021 [hep-th/9812031].

[5] O. Bergman and M.R. Gaberdiel, *Non-BPS states in heterotic-type IIA duality*, *J. High Energy Phys.* **03** (1999) 013 [hep-th/9901014].

[6] M. Gutperle, *Non-BPS D-branes and enhanced symmetry in an asymmetric orbifold*, *J. High Energy Phys.* **08** (2000) 036 [hep-th/0007126].

[7] A. Sen, SO(32) *spinors of type I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

[8] A. Sen, *Type I D-particle and its interactions*, *J. High Energy Phys.* **10** (1998) 021 [hep-th/9809111].

[9] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, *Stable non-BPS D-branes in type I string theory*, *Nucl. Phys.* **B 564** (2000) 60 [hep-th/9903123].

[10] O. Bergman, E.G. Gimon and P. Hořava, *Brane transfer operations and T-duality of non-BPS states*, *J. High Energy Phys.* **04** (1999) 010 [hep-th/9902160].

[11] T. Dasgupta and B. Stefański Jr., *Non-BPS states and heterotic–type I' duality*, *Nucl. Phys.* **B 572** (2000) 95 [hep-th/9910217].

[12] E. Witten, *D-branes and K-theory*, *J. High Energy Phys.* **12** (1998) 019 [hep-th/9810188].

[13] I. Brunner and J. Distler, *Torsion D-branes in nongeometrical phases*, hep-th/0102018.

[14] V. Braun, *K-theory torsion*, hep-th/0005103.

[15] O. Loaiza-Brito and A.M. Uranga, *The fate of the type I non-BPS D7-brane*, hep-th/0104173.

[16] D.-E. Diaconescu, G. Moore and E. Witten, E_8 *gauge theory and a derivation of K-theory from M-theory*, hep-th/0005090.

[17] D.-E. Diaconescu, G. Moore and E. Witten, *A derivation of K-theory from M-theory*, hep-th/0005091.

[18] J. de Boer, R. Dijkgraaf, K. Hori, A. Keurentjes, J. Morgan, D.R. Morrison and S. Sethi, *Triples, fluxes and strings*, hep-th/0103170.

[19] C. Vafa and E. Witten, *Dual string pairs with* $N = 1$ *and* $N = 2$ *supersymmetry in four dimensions*, *Nucl. Phys.* **46** *(Proc. Suppl.)* (1996) 225 [hep-th/9507050].

[20] A. Dabholkar and J. Park, *Strings on orientifolds*, *Nucl. Phys.* **B 477** (1996) 701 [hep-th/9604178].

[21] E. Witten, *Toroidal compactification without vector structure*, *J. High Energy Phys.* **02** (1998) 006 [hep-th/9712028].

[22] M. Bianchi, *A note on toroidal compactifications of the type I superstring and other superstring vacuum configurations with 16 supercharges*, *Nucl. Phys.* **B 528** (1998) 73 [hep-th/9711201].

[23] P. Hořava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209].

[24] M.R. Gaberdiel and B. Stefański Jr., *Dirichlet branes on orbifolds*, *Nucl. Phys.* **B 578** (2000) 58 [hep-th/9910109].

[25] S. Gukov, *K-theory, reality and orientifolds*, *Comm. Math. Phys.* **210** (2000) 621 [hep-th/9901042].

[26] O. Bergman and M.R. Gaberdiel, *Dualities of type 0 strings*, *J. High Energy Phys.* **07** (1999) 022 [hep-th/9906055].

[27] L. Houart and Y. Lozano, *Brane descent relations in M-theory*, *Phys. Lett.* **B 479** (2000) 299 [hep-th/0001170].

[28] K. Intriligator, M. Kleban and J. Kumar, *Comments on unstable branes*, *J. High Energy Phys.* **02** (2001) 023 [hep-th/0101010].

[29] R. Bott and T. Tu, *Differential forms in algebraic topology*, Springer, GTM, 1982.

[30] S. Chaudhuri, G. Hockney and J.D. Lykken, *Maximally supersymmetric string theories in $d < 10$*, *Phys. Rev. Lett.* **75** (1995) 2264 [hep-th/9505054].

[31] O. Bergman, E.G. Gimon and B. Kol, *Strings on orbifold lines*, *J. High Energy Phys.* **05** (2001) 019 [hep-th/0102095].

[32] M. Atiyah, *K-theory*, W.A. Benjamin, 1964.

[33] J.F. Adams, *Vector fields on spheres*, *Ann. of Math.* **75** (1962) 603.

[34] M. Karoubi, *K-theory: an introduction*, Springer, 1978.